

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Thursday 19 June 2025 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours

plus your additional time allowance

YOU MUST HAVE:

**the Printed Answer Booklet or any suitable paper
provided by the centre. The Printed Answer Booklet may
be enlarged by the centre.**

a scientific or graphical calculator

**Insert for Questions 3(a), 8, 10 and 12(a) (with
this document)**

READ INSTRUCTIONS OVERLEAF



INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

If you use the Printed Answer Booklet write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

If you use the Printed Answer Booklet fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.

The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 100.

The marks for each question are shown in brackets [].

ADVICE

Read each question carefully before you start your answer.

FORMULAE

A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$$

Geometric series

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \quad \text{for } |r| < 1$$

Binomial series

$$(a + b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$$

$(n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$

$$(|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}, \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x) (f(x))^n dx = \frac{1}{n+1} (f(x))^{n+1} + c$$

Integration by parts $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$

Small angle approximations

$\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{1}{2}\theta^2$, $\tan \theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi \right)$$

Numerical methods

Trapezium rule:

$$\int_a^b y \, dx \approx \frac{1}{2}h \left\{ (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \right\},$$

$$\text{where } h = \frac{b - a}{n}$$

The Newton-Raphson iteration for solving

$$f(x) = 0: x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B | A) = P(B)P(A | B)$$

$$\text{OR } P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum (x - \bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{OR} \quad \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$,

mean of X is np , variance of X is $np(1 - p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995
z	0.674	1.282	1.645	1.960	2.326	2.576

p	0.9975	0.999	0.9995
z	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A

Pure Mathematics

- 1 (a) Express $3x^2 - 12x + 17$ in the form $a(x - b)^2 + c$ where a , b and c are constants. [3]
- (b) State the coordinates of the minimum point of the curve $y = 3x^2 - 12x + 17$. [1]
- (c) State the equation of the normal to the curve $y = 3x^2 - 12x + 17$ at its minimum point. [1]
- 2 (a) The curve $y = \frac{1}{x^3}$ is translated by 4 units in the positive x -direction.

Write down the equation of the curve after it has been translated. [2]

- (b) The curve $y = 5x^2$ is stretched parallel to the y -axis with scale factor 4.

The point on the curve $y = 5x^2$ with x -coordinate 3 is transformed to the point P .

Write down the coordinates of P . [2]

- 3 (a) Sketch the graph of $y = |2x - 5|$ on the grid provided in the Printed Answer Booklet or in the Insert. [2]

(b) Solve the inequality $|2x - 5| < 1$. [2]

(c) In this question you must show detailed reasoning.

Hence find ALL the possible integer values N that satisfy the inequality

$$|2e^{0.1N} - 5| < 1. \quad [3]$$

- 4 Functions f and g are defined for all real values of x by

$f(x) = \frac{x-k}{2}$ and $g(x) = x^2 + kx + 5$, where k is a constant.

You are given that the equation $f^{-1}g(x) = 4 - 2kx$ has real distinct roots.

(a) Show that k satisfies the inequality $2k^2 - k - 6 > 0$. [5]

(b) In this question you must show detailed reasoning.

Hence find the set of values of k . Give your answer in set notation. [3]

- 5 (a) Use the substitution $u = \cos x$ to show that

$$\int \frac{1 + \cos x}{\sin x} dx = \ln(1 - \cos x) + c. \quad [4]$$

FIG. 1

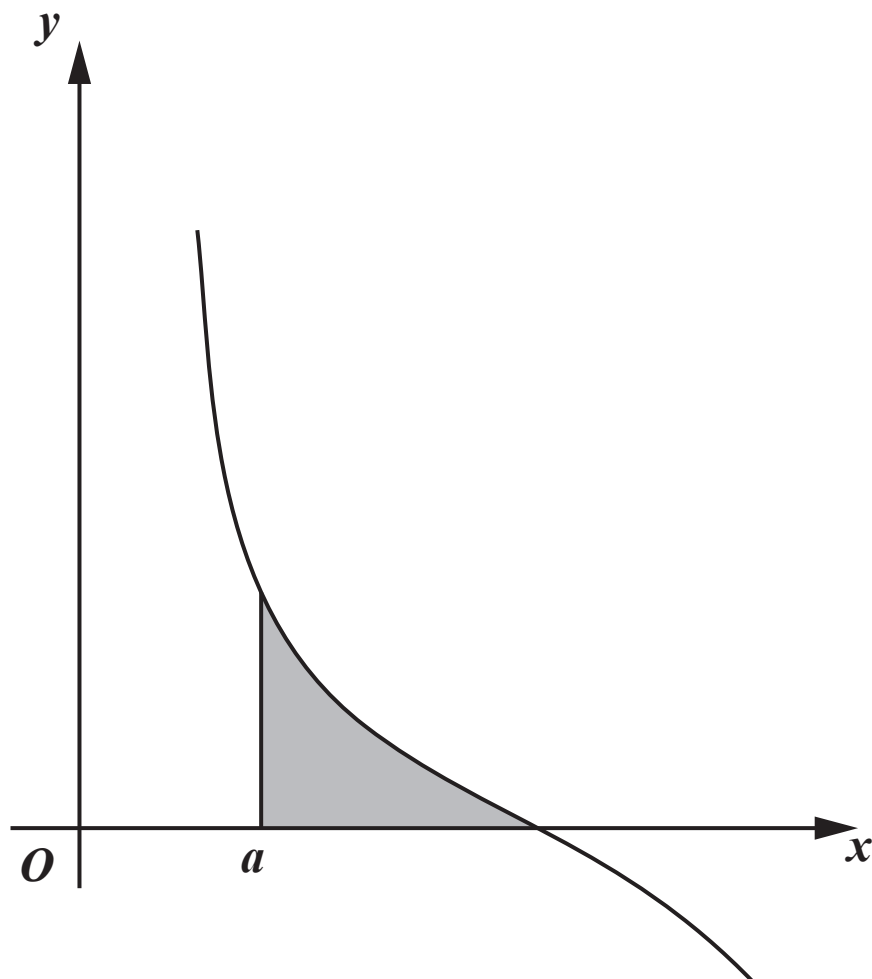


FIG. 1 shows part of the curve $y = \frac{1 + \cos x}{\sin x}$.

The shaded region is bounded by the curve, the x -axis, and the line $x = a$.

You are given that the area of the shaded region is a square units.

- (b) Show that the value of a satisfies the equation $a - \cos^{-1}(1 - 2e^{-a}) = 0$. [4]

(c) Show by calculation that the value of a lies between 1.1 and 1.2. [2]

(d) Use the iterative formula

$$a_{n+1} = \cos^{-1}(1 - 2e^{-a_n}),$$

with starting value $a_1 = 1.2$, to find the value of a correct to 2 decimal places. Show the result of each step of the iterative process. [2]

FIG. 2

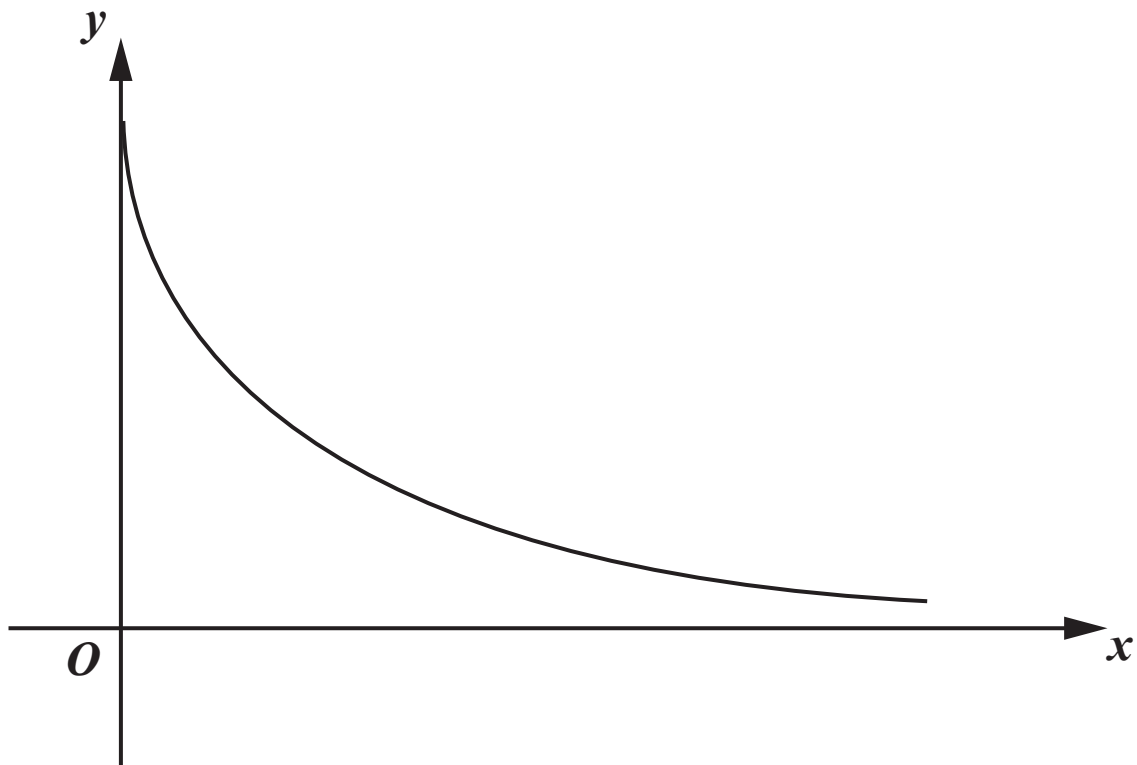


FIG. 2 shows the curve $y = \cos^{-1}(1 - 2e^{-x})$.

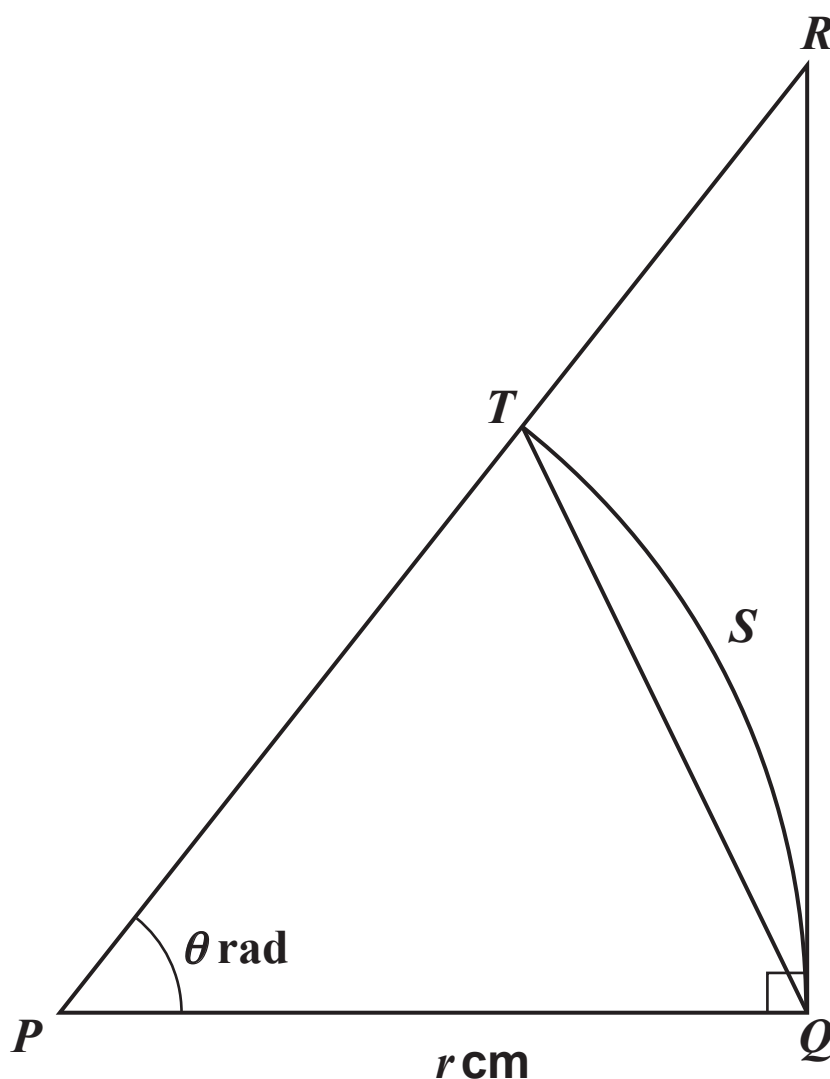
(e) Explain why the gradient of the curve $y = \cos^{-1}(1 - 2e^{-x})$ at the point where $x = a$, where a is the value found in part (d), lies in the interval $(-1, 0)$. [2]

6 The compound angle formulae for $\sin(A + B)$ and $\sin(A - B)$ are

**$\sin(A + B) = \sin A \cos B + \cos A \sin B$ and
 $\sin(A - B) = \sin A \cos B - \cos A \sin B$.**

(a) By letting $C = A + B$ and $D = A - B$, show that

$$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right). \quad [1]$$



The diagram shows a right-angled triangle PQR with $PQ = r \text{ cm}$. The angle QPR is θ radians. The diagram also shows the sector $PQST$ of a circle with centre P and radius $r \text{ cm}$. The line segment QT is a chord of the sector $PQST$.

(b) By considering the areas of triangle PQT , sector $PQST$ and triangle PQR , show that

$$1 < \frac{\theta}{\sin \theta} < \frac{1}{\cos \theta}. \quad [4]$$

(c) (i) Hence write down a similar inequality interval for the expression $\frac{\sin \theta}{\theta}$. [1]

(ii) Hence state $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$. [1]

(d) Using parts (a) and (c)(ii), show from first principles that the derivative of $\sin x$ is $\cos x$, where x is measured in radians. [4]

A student attempts to use the result regarding the derivative of $\sin x$ to find the derivative of $\cos x$. The student's attempt is shown below.

Let $y = \cos x$, where x is measured in radians.

$$y = \sin\left(\frac{\pi}{2} - x\right)$$

so $\frac{dy}{dx} = \cos\left(\frac{\pi}{2} - x\right)$

but $\sin x \equiv \cos\left(\frac{\pi}{2} - x\right)$

therefore $\frac{dy}{dx} = \sin x$.

(e) Identify the error made by the student. [1]

Section B

Mechanics

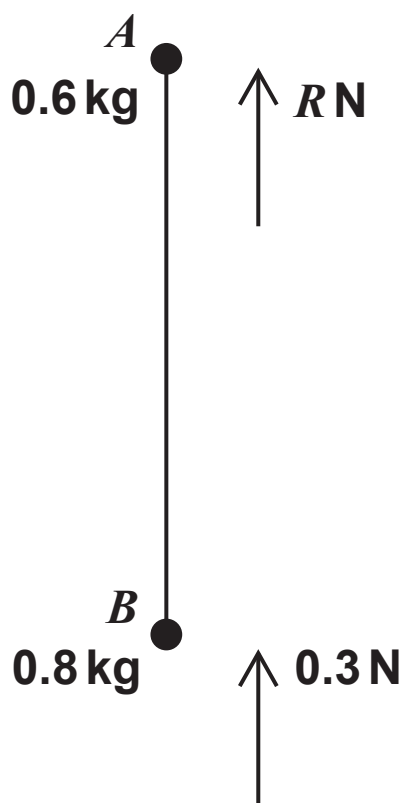
- 7 A particle P of mass 3 kg is moving on a smooth horizontal surface under the action of two constant horizontal forces $(5\mathbf{i} + 3\mathbf{j})\text{ N}$ and $(a\mathbf{i} + 3b\mathbf{j})\text{ N}$. The acceleration of P is $(2\mathbf{i} - 3\mathbf{j})\text{ m s}^{-2}$.

(a) Find the value of a and the value of b . [3]

At time $t = 0$ seconds the velocity of P is $\underline{u}\text{ m s}^{-1}$ and at time $t = 5$ seconds the velocity of P is $(7\mathbf{i} - 6\mathbf{j})\text{ m s}^{-1}$.

(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, an expression for $\underline{u}$. [2]

- 8 There is a copy of the diagram in the insert if you need to use it.



Two particles A and B of masses 0.6 kg and 0.8 kg respectively, are attached to the ends of a light inextensible string. Particle A is held at rest at a fixed point and B hangs vertically below A .

Particle A is now released. As the particles fall the air resistance acting on A has a constant magnitude of $R\text{ N}$ and the air resistance acting on B has a constant magnitude of 0.3 N . The string remains taut throughout the subsequent motion (see diagram).

The downward acceleration of each of the particles is 9.2 m s^{-2} and the tension in the string is $T\text{ N}$.

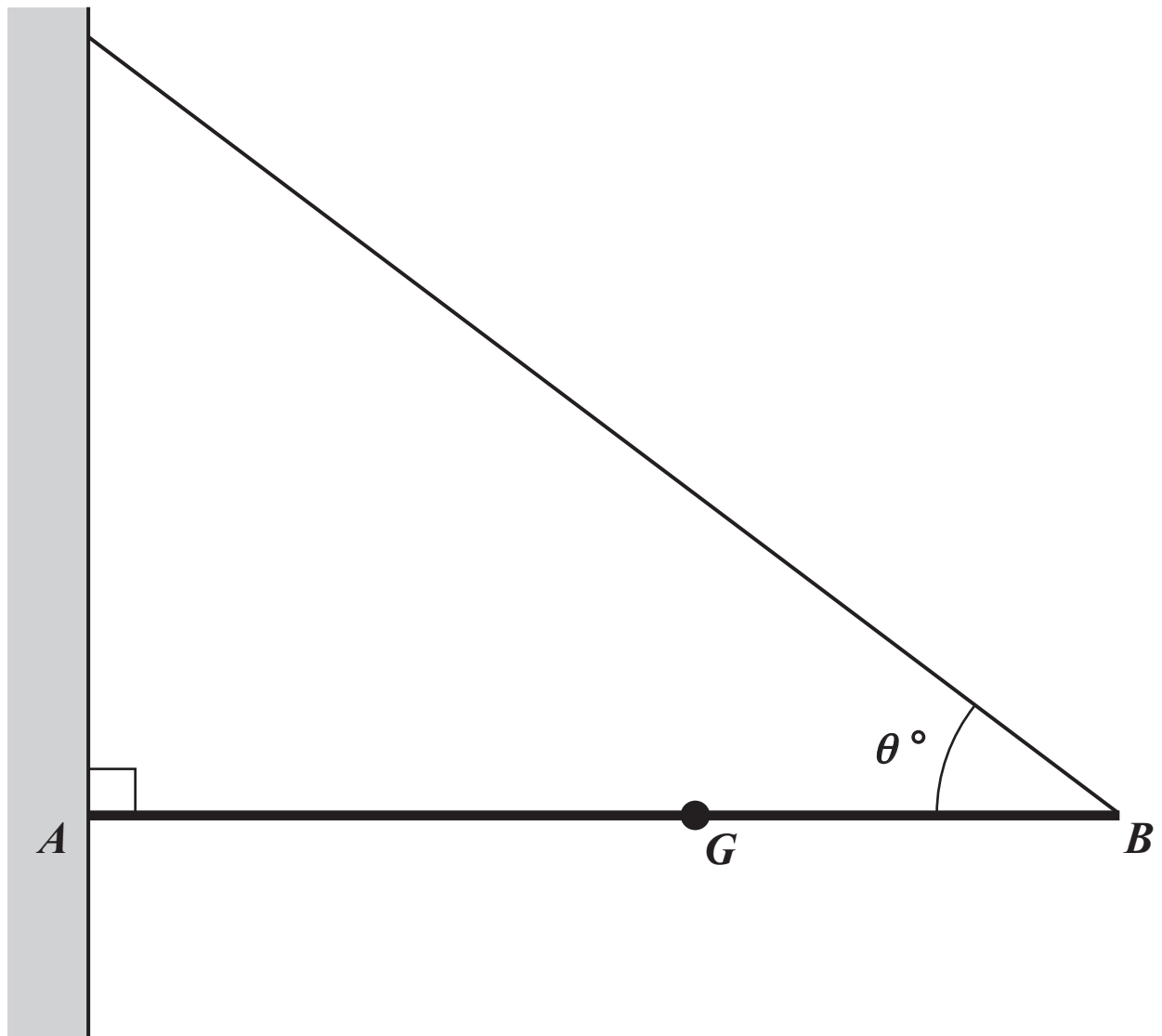
- (a) By applying Newton's second law to B , find the value of T . [2]
- (b) Hence, or otherwise, find the value of R . [2]

9 In this question you must show detailed reasoning.

A particle moves in a straight line. Its velocity t seconds after leaving a fixed point on the line is $v \text{ m s}^{-1}$ where $v = 2 + 3t + 0.6t^2$.

Find the distance travelled by the particle from time $t = 0$ until the instant when its acceleration is 15 m s^{-2} .
[6]

10 There is a copy of the diagram in the insert if you need to use it.



The diagram shows a NON-UNIFORM rod AB of mass 4 kg and length 2 m. The end A of the rod rests against a rough vertical wall. The rod is held in a horizontal position, perpendicular to the wall, by a light inextensible string attached to the rod at B . The other end of the string is attached to the wall at a point vertically above A . The string is inclined at an angle of θ° to the horizontal, where $\sin \theta = \frac{4}{5}$.

The rod rests in equilibrium in a vertical plane perpendicular to the wall. The rod's weight acts at the point G on AB .

You are given that the magnitude of the moment of the rod's weight about A is 47.04 N m.

- (a) Show that the distance AG is 1.2 m. [1]
- (b) Find the tension in the string. [2]
- (c) Determine the magnitude of the contact force exerted on the rod by the wall at A . [5]

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- 11** In this question you should take the acceleration due to gravity to be 10 m s^{-2} .

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are in a vertical plane with $\mathbf{i}$ horizontal and $\mathbf{j}$ vertically upwards.

A small ball P is projected from a point O on horizontal ground into the air. When P is in the air, it is to be modelled as a particle moving under the influence of gravity only.

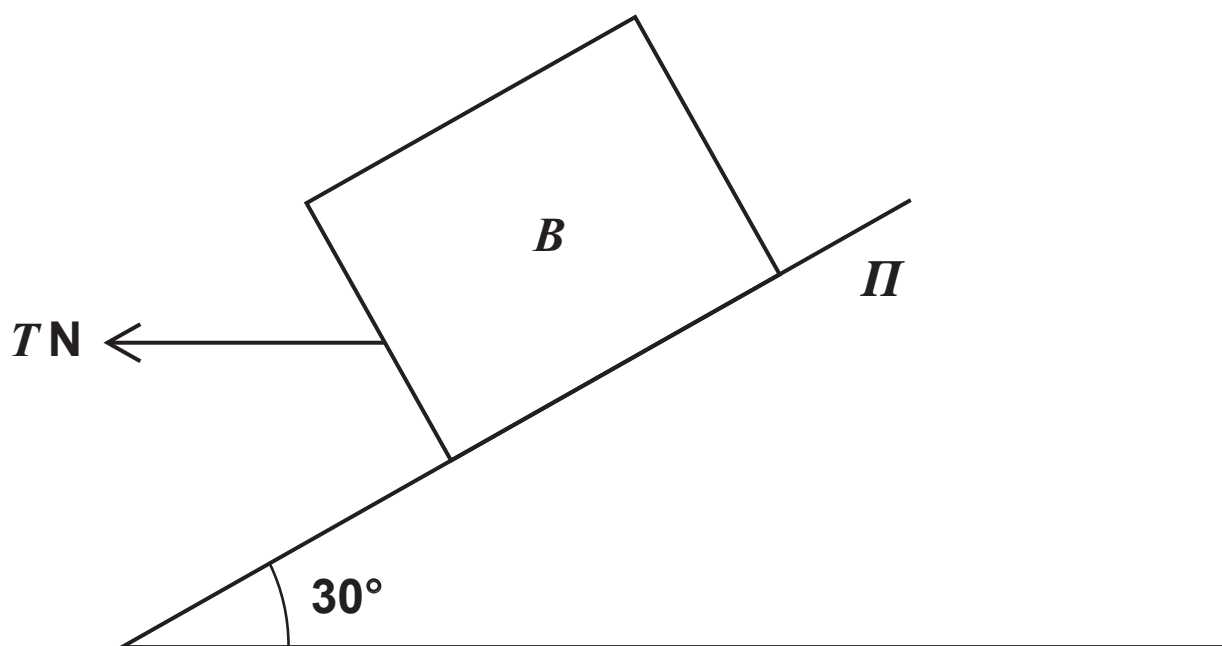
At time $t = 2$ seconds, P has velocity $(2\mathbf{i} - 8\mathbf{j}) \text{ m s}^{-1}$. The position vector of a point on the trajectory of P is $(x\mathbf{i} + y\mathbf{j})\text{m}$ relative to O .

- (a) Show that $y = 6x - 1.25x^2$. [5]
- (b) Determine the maximum height of P above the ground during its motion. [3]
- (c) Determine the following as P passes through the point on its trajectory where $x = 2.5$.
- the speed of P
- the direction of motion of P [5]

In reality P does not move only under the influence of gravity but is also subject to air resistance.

- (d) Explain how this would affect your answer to part (b). [1]

FIG. 1



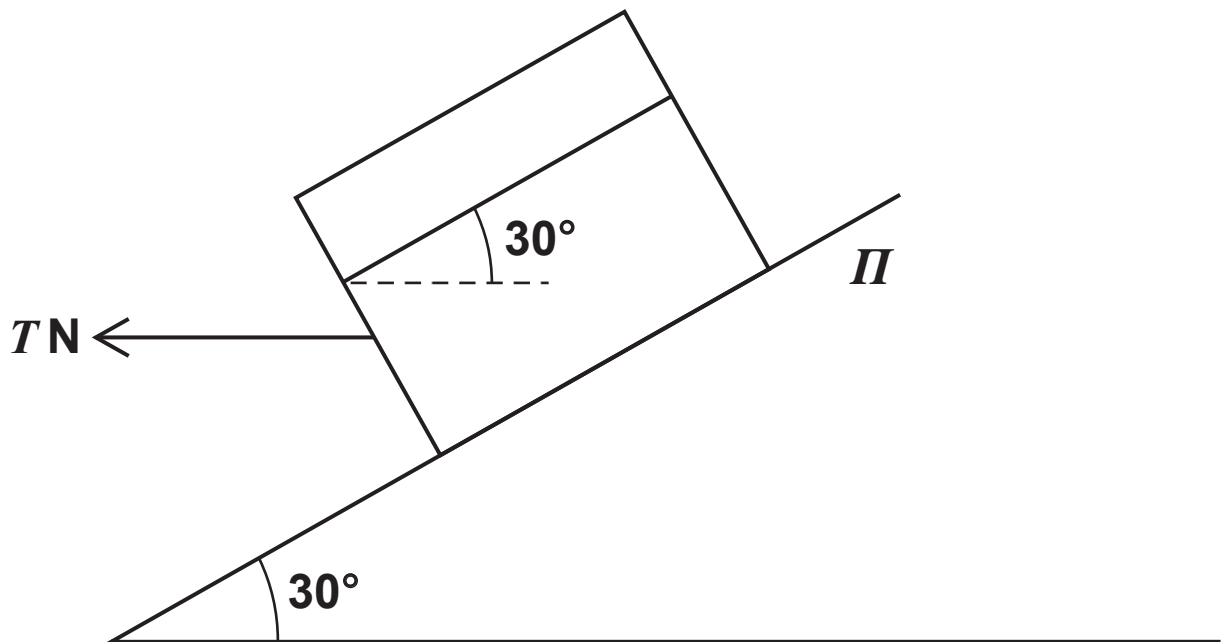
A rectangular block B of mass 10 kg lies at rest in limiting equilibrium on a rough plane Π inclined at 30° to the horizontal. A horizontal force of magnitude $T \text{ N}$, acting above a line of greatest slope, is applied to B (see FIG. 1).

The coefficient of friction between B and the plane is 0.8 .

- (a) Show that the value of T is 14.9 , correct to 3 significant figures. There is a copy of the diagram in the insert if you need to use it. [6]

For the remainder of the question, you should use this value of T .

FIG. 2



Block B is now cut at an angle of 30° to the horizontal into two smaller blocks. The upper block has a mass of 4 kg , and the lower block has a mass of 6 kg . The two blocks are held at rest with the lower block on Π . The horizontal force of magnitude $T\text{ N}$ is now applied to the lower block (see FIG. 2).

The two blocks are released from rest and in the subsequent motion the upper block starts to move with acceleration 3.5 m s^{-2} .

- (b) Determine the coefficient of friction between the two blocks. [4]
- (c) Show that the lower block does NOT move. [3]

END OF QUESTION PAPER

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